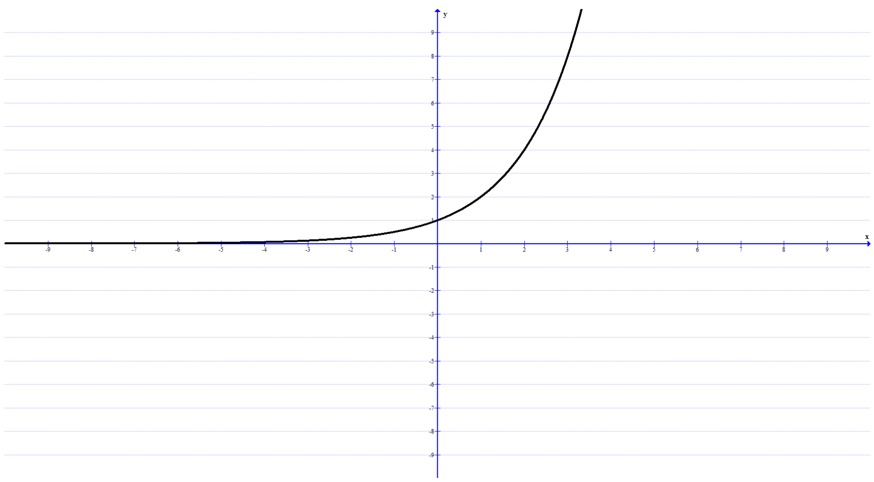
Section 6.3: Exponential functions

1) let $f(x) = 2^x$

1a) make a table of values and sketch a graph

x	f(x)
2	$2^2 = 4$ (2,4)
1	$2^1 = 2$ (1,2)
0	$2^0 = 1$ (0,1)
-1	$2^{-1} = ½$ (-1, ½)
-2	$2^{-2} = ¼$ (-2, ¼)



1b) Find the domain of f(x)

The graph needs to be extended all the way to the left and right edge of the x-axis.

Answer: domain $(-\infty, \infty)$

1c) Find the range of f(x)

The graph is supposed to always be just slightly above the x-axis. So the bottom y for the range is 0. I use a round bracket as the graph doesn't touch the x-axis. The graph goes up forever, so the end of the range is ∞

Answer: $\boldsymbol{range}\ (\mathbf{0}, \infty)$

1d) Find the horizontal asymptote

The x-axis is the horizontal asymptote

Answer: y = 0

3) let $f(x) = 2^{x+3}$

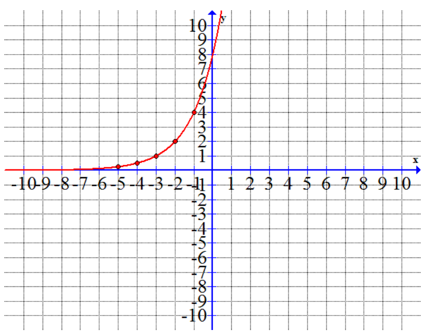
3a) make a table of values and sketch a graph

Set the exponent equal to zero to get the (-3) for the middle of the x-column of the table.

x + 3 = 0

x = -3

x	f(x)
-5	$2^{-5+3} = 2^{-2} = =1/2^2 = 1/4$ (-5, 1/4)
-4	$2^{-4+3} = 2^{-1} = ½$ (-4, ½)
-3	$2^{-3+3} = 2^0 = 1$ (-3, 1)
-2	$2^{-2+3} = 2$ (-2, 2)
-1	$2^{-1+3} = 2^2 = 4$ (-1, 4)



3b) Find the domain of f(x)

the graph needs to be extended all the way to the left and right edges of the x-axis.

Answer: domain $(-\infty, \infty)$

3c) Find the range of f(x)

The graph is supposed to always be just slightly above the x-axis. So the bottom y for the range is 0. I use a round bracket as the graph doesn't touch the x-axis. The graph goes up forever, so the end of the range is ∞

Answer: $\boldsymbol{range\ (0, \infty)}$

3d) Find the horizontal asymptote

The x-axis is the horizontal asymptote

Answer: y = 0

5) f(x – 3)

5a) To find f(x-3) just change the exponent from x to x – 3. You really don't need a parenthesis in your answer.

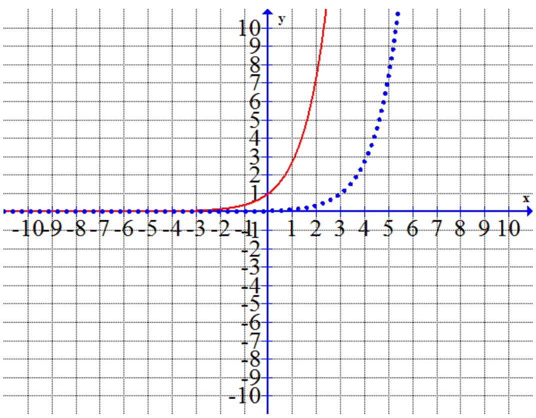
Answer #5a: f(x – 3) = e^{x-3}

change the sign of the number in the exponent, put that number in the middle of the x-column

5b) Since the minus three is in a parenthesis the graph will shift to the right.

Answer #5b: shift right 3 units

5c) Create the graph by moving each point 3 to the right.
f(x – 3) is drawn in blue.



5d) Find the domain of f(x)

the graph needs to be extended all the way to the left and right edges of the x-axis.

Answer #5d: domain $(-\infty, \infty)$
5e) Find the range of f(x)

The graph is supposed to always be just slightly above the x-axis. So the bottom y for the range is 0. I use a round bracket as the graph doesn't touch the x-axis. The graph goes up forever, so the end of the range is ∞

Answer #5e: $\boldsymbol{range}\ (\mathbf{0}, \infty)$
5f) Find the horizontal asymptote

The x-axis is the horizontal asymptote

Answer #5f: y = 0

7a) To find f(x + 2) just change the x in the exponent to x + 2, no parenthesis is needed.

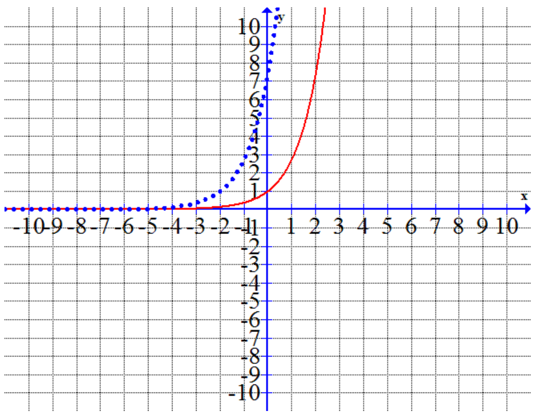
Answer #7a: f(x + 2) = e^{x+2}

7b) Describe the transformation from f(x) = e^x

The minus in a parenthesis will shift the graph left.

Answer #7b: The graph is the same, but shifted 2 units to the left.

7c) Just move each point 2 to the left and duplicate the original graph.
f(x + 2) drawn in blue.



7d) Find the domain of f(x)

the graph needs to be extended all the way to the left and right edges of the x-axis.

Answer #7d: domain $(-\infty, \infty)$
7e) Find the range of f(x)

The graph is supposed to always be just slightly above the x-axis. So the bottom y for the range is 0. I use a round bracket as the graph doesn't touch the x-axis. The graph goes up forever, so the end of the range is ∞

Answer #7e: $\boldsymbol{range}\ (\mathbf{0}, \infty)$
7f) Find the horizontal asymptote

The x-axis is the horizontal asymptote

Answer #7f: y = 0

9) f(x) + 2

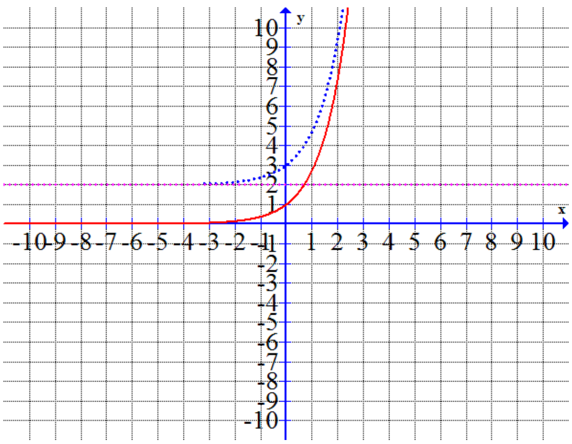
9a) simply add 2 to the function. The 2 does not belong in the exponent as it is not in a parenthesis.

Answer #9a: f(x) + 2 = e^x + 2

9b) The plus 2 is not part of the function notation, so it moves the graph up 2.

Answer #9b: shifts graph up 2 units.

9c) Just move each point of the original graph up two units and draw the same shape.
f(x) + 2 is drawn in blue and the horizontal asymptote is drawn in purple.



9d) Find the domain of f(x)

the graph needs to be extended all the way to the left and right edges of the x-axis.

Answer #9d: domain $(-\infty, \infty)$
9e) Find the range of f(x)

The graph is raised up 2. So the bottom y for the range is 2. I use a round bracket as the graph doesn't touch the x-axis. The graph goes up forever, so the end of the range is ∞

Answer #9e: $\boldsymbol{range\ (2, \infty)}$
9f) Find the horizontal asymptote

The graph bottoms out at 2 on the y-axis

Answer #9f: y = 2

11) f(x) – 3

11a) simply subtract 3 after the e^x. The 3 does not belong in the exponent as it is not in a parenthesis.

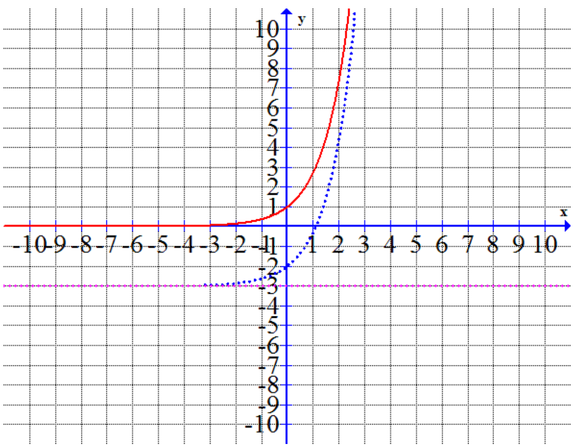
Answer #11a: f(x) – 3 = e^x – 3

11b) The -3 moves the graph down.

Answer #11b: shifts graph down 3 units.

11c) Just move each point down 3 units and draw the same shape.

Graph of f(x) – 3 drawn in blue, horizontal asymptote drawn in purple.



11d) Find the domain of f(x)

the graph needs to be extended all the way to the left and right edges of the x-axis.

Answer #11d: domain $(-\infty, \infty)$

11e) Find the range of f(x)

The graph moves down 3, so the range also moves down 3. New range starts at y = -3

Answer #11e: $\boldsymbol{range}\ (-\mathbf{3}, \infty)$

11f) The horizontal asymptote moves down 3 as well.

Answer #11f: y = -3

13) f(-x)

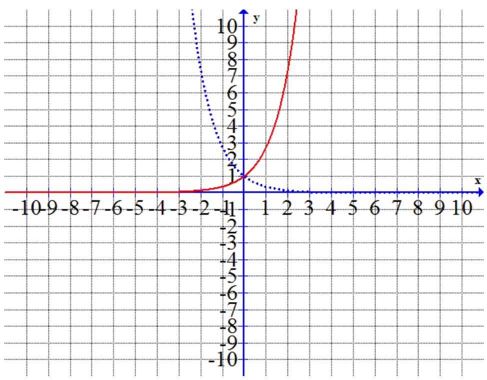
13a) since the negative is in a parenthesis, the negative x will go in the exponent.

Answer #13a: f(-x) = e^{-x}

13b) f(-x) reflects a graph over the y-axis.

Answer #13b: Reflects over y-axis.

13c) Just move each point to the opposite side of the y-axis.
f(-x) drawn in blue.



13d) Find the domain of f(x)

the graph needs to be extended all the way to the left and right edges of the x-axis.

Answer #13d: domain $(-\infty, \infty)$

13e) Find the range of f(x)

The graph is supposed to always be just slightly above the x-axis. So the bottom y for the range is 0. I use a round bracket as the graph doesn't touch the x-axis. The graph goes up forever, so the end of the range is ∞

Answer #13e: $\boldsymbol{range\ (0, \infty)}$

13f) Find the horizontal asymptote

The horizontal asymptote will correspond with the range.

Answer #13f: y = 0

15a) To find 2f(x) simply put a 2 in front of the original function.

Answer #15a: 2f(x) = 2e^{x}

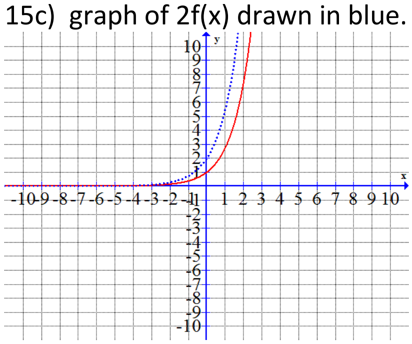
15b) Each point on the new graph will have the same x as the original, but the y's will be multiplied by 2, and be pulled away from the y-axis. We say that the points are stretched away from the y-axis.

Answer #15b: new graph will be stretched.

15c) We need to do more than shift to create the graph of 2f(x) as this is not a rigid transformation. The easiest way to create a graph is to just multiply each y in the original table by 2 to get the y-coordinate for the same value of x in the new function.

The first two columns in my table were the given columns. I multiplied each y coordinate by 2 to get the middle column. The last column are the points that I plotted to get the graph of 2f(x)

15c) graph of 2f(x) drawn in blue.



Multiply each y in the original table of f(x) by 2 to get the y-values for the points in 2f(x)

x	f(x)	2f(x) y computation	Point on graph of 2f(x)
-2	$\frac{1}{e^2}$ = 0.14	2*0.14 = 0.28	(-2, 0.28)
-1	$\frac{1}{e}$ =0.37	2*0.37 = 0.74	(-1, 0.74)
0	1	2*1 = 2	(0, 2)
1	e = 2.72	2*2.72 = 5.44	(1, 5.44)
2	e^2 = 7.39	2*7.39 = 14.78	(2, 14.78)

15d) Find the domain of f(x)

the graph needs to be extended all the way to the left and right edges of the x-axis.

Answer #15d: domain $(-\infty, \infty)$

15e) Find the range of f(x)

The graph is supposed to always be just slightly above the x-axis. So the bottom y for the range is 0. I use a round bracket as the graph doesn't touch the x-axis. The graph goes up forever, so the end of the range is ∞

Answer #15e: $\boldsymbol{range\ (0, \infty)}$

15f) Find the horizontal asymptote

The horizontal asymptote will correspond with the range.

Answer #15f: y = 0

#17 - 32 Let $g(x) = 2^x$

a) Find the indicated function

b) Describe the transformation of the graph of as compared to the graph of $g(x) = 2^x$.

17) g(x+1)

17a) Find the indicated function

The plus 1 goes in the exponent. It's okay if you put (x+1) in a parenthesis, but the parenthesis are not necessary. (**This shifts the graph left 1)**

Answer #17a: $g(x + 1) = 2^{x+1}$

17b) Describe the transformation of the graph of as compared to the graph of $g(x) = 2^x$.

Answer #17b: Shifts left 1

19) g(x-1)

19a) Find the indicated function

The minus 1 goes in the exponent. It's okay if you put (x-1) in a parenthesis, but the parenthesis are not necessary. **(This shifts the graph right 1)**

Answer #19a: $g(x – 1) = 2^{x-1}$

19b) Describe the transformation of the graph of as compared to the graph of $g(x) = 2^x$.

Answer #19b: Shifts right 1

21) g(x) + 1

21a) Find the indicated function

The plus 1 doesn't belong in the exponent as it is not in a parenthesis. It just goes after the function. **(This shifts the graph up 1)**

Answer #21a: $g(x) + 1 = 2^x + 1$

21b) Describe the transformation of the graph of as compared to the graph of $g(x) = 2^x$.

Answer #21b: Shifts up 1

23) g(x) - 2

23a) Find the indicated function

The minus 2 doesn't belong in the exponent as it is not in a parenthesis. It just goes after the function. **(This shifts the graph down 2)**

Answer #23a: g(x) - 2 = 2^x - 2

23b) Describe the transformation of the graph of as compared to the graph of g(x) = 2^x.

Answer #23b: shifts down 2

25) –g(x)

25a) Find the indicated function

The negative sign will just go to the left of the 2. It doesn't belong in the exponent as it is not in the parenthesis. **(This reflects the graph over the x-axis)**

Answer #25a: -g(x) = -2^x

25b) Describe the transformation of the graph of as compared to the graph of g(x) = 2^x.

Answer #25b: reflects over x-axis

27) g(x+1) - 4

27a) Find the indicated function

The plus 1 goes in the exponent. It's okay if you put (x+1) in a parenthesis, but the parenthesis are not necessary. **(This shifts the graph left 1)**

The minus 4 doesn't belong in the exponent as it is not in a parenthesis. It just goes after the function. **(This shifts the graph down 4)**

Answer #27a: g(x + 1) – 4 = 2^{x+1} - 4

27b) Describe the transformation of the graph of as compared to the graph of g(x) = 2^x.

Answer #27b: Shifts left 1 and down 4

29) g(x – 2) + 3

29a) Find the indicated function

The minus 2 goes in the exponent. It's okay if you put (x-2) in a parenthesis, but the parenthesis are not necessary. **(This shifts the graph right 2)**

The plus 3 doesn't belong in the exponent as it is not in a parenthesis. It just goes after the function. **(This shifts the graph up 3)**

Answer #29a: g(x – 2) + 3 = 2^{x-2} + 3

29b) Describe the transformation of the graph of as compared to the graph of g(x) = 2^x.

Answer #29b: shifts right 2 and up 3

31) –g(-x) + 2

31a) Find the indicated function

The negative sign in front of the g(x) will just go to the left of the 2. It doesn't belong in the exponent as it is not in the parenthesis. **(This reflects over x-axis)**

The negative sign inside the parenthesis will go in the exponent. **(This reflects over y-axis)**

The plus 2 doesn't belong in the exponent as it is not in a parenthesis. It just goes after the function. **(This shifts graph up 2)**

Answer #31a: -g(-x) + 2 = -2^{-x} + 2

31b) Describe the transformation of the graph of as compared to the graph of g(x) = 2^x.

Answer #31b: reflects over x-axis, reflects over y-axis, shifts up 2

33) $3^{x+2} = 27$

$3^{x+2} = 3^3$

x + 2 = 3

Answer #33: x = 1

35) $4^{-x} = 64$

$4^{-x} = 4^3$

-x = 3

Answer #35: x = -3

37) $\left(\frac{1}{2}\right)^{x+3} = \frac{1}{32}$

$(2^{-1})^{x+3} = 2^{-5}$

Clear the parenthesis on the left side by multiplying the exponents.

-1*x and -1*3

$2^{-1x-3} = 2^{-5}$

-1x – 3 = -5
+3 +3

-1x = - 2

x = 2

Answer #37: x = 2

39) $\left(\frac{1}{2}\right)^{2x+1} = 16$

$(2^{-1})^{2x + 1} = 2^4$

Multiply exponents to clear parenthesis

-1*2x and -1*1

$2^{-2x-1} = 2^4$

-2x – 1 = 4
<u>+1 +1</u>
-2x = 5

x = -5/2

Answer #39: x = -5/2

41) $5^{x+2} = 25^{3x-4}$
$5^{x+2} = (5^2)^{3x-4}$
$5^{x + 2} = 5^{6x - 8}$

x + 2 = 6x – 8
<u>-x +8 -x + 8</u>
10 = 5x

Answer #41: x = 2

43) $3^{x-4} = 9^{3-x}$

$3^{x-4} = (3^2)^{3-x}$

$3^{x - 4} = 3^{6 - 2x}$

x – 4 = 6 – 2x
<u>-x -6 - 6 -x</u>
-10 = -3x
-10/-3 = x

Answer #43: x = 10/3

45) $e^{x-4} = e^{2x-3}$

x – 4 = 2x – 3
-x +3 - x + 3
-1 = x

Answer #45: x = -1

47) $e^{3x} * e^{2} = e^{-4}$

Add exponents to simplify left side.

$e^{3x+2} = e^{-4}$

3x + 2 = -4
- 2 - 2
3x = -6

Answer #47: x = -2

49) $2^{x-4} * 2^{3} = 2^{5}$

Add exponents to simplify left side.

$2^{x-4+3} = 2^{5}$

$2^{x-1} = 2^{5}$

x – 1 = 5

Answer #49: x = 6